

Integrabilities of the $t - J$ Model with Impurities

Zhan-Ning Hu^{a,b,*} and Fu-Cho Pu^{b,c,†}

^aCCAST(World Lab.), P.O.Box 8730, Beijing 100080

^bInstitute of Physics, Academia Sinica, Beijing 100080, China[‡]

^cDepartment of Physics, Guangzhou teacher colleague, Guangzhou 510400, China

Yupeng Wang[§]

Cryogenic Laboratory, Academia Sinica, Beijing 100080, China

Abstract

The hamiltonian with magnetic impurities coupled to the strongly correlated electron system is constructed from $t - J$ model. And it is diagonalized exactly by using the Bethe ansatz method. Our boundary matrices depend on the spins of the electrons. The Kondo problem in this system is discussed in details. The integral equations are derived with complex rapidities which describe the bound states in the system. The finite-size corrections for the ground-state energies are obtained.

PACS: 0550; 7127; 7420

Keywords: Strongly correlated electron system; $t - J$ model; Bethe ansatz equations; Boundary matrices; Kondo problem; Ground-state energy; Finite-size correction

*E-mail: huzn@aphy01.iphy.ac.cn

†E-mail: pufc@aphy02.iphy.ac.cn

‡ mail address

§E-mail: ypwang@cl.cryo.ac.cn

I. INTRODUCTION

The Kondo problem devoted to study the effect due to the exchange interaction between the impurity spin and the electron gas has played an important role in condensed matter physics since its discovery [1]. The original treatments in Kondo problem the electron-electron interaction is discarded. This is reasonable in 3D where the interacting electron system can be described by Fermi liquid. Recently, much attention has been paid to the theory of the magnetic impurities in the Fermi liquid and Luttinger liquid, [2], [3] where the central scheme is the few impurity coupled with strongly-correlated electron system. Apart from the fundamental theoretical interests, it is remarkable that the physics implied here can be accessible experimentally. The recent advances in semiconductor technology enable to fabricate very narrow quantum wire which can be considered one-dimensional, and furnishes a real system of Luttinger liquid. Also edge states in a 2D electron gas for fractional quantum Hall effect can be considered as Luttinger liquid [4]. Intense efforts and much progress has been made around the subjects from different approaches. Using bosonization and renormalization techniques, Kane and Fisher [5] studied transport of a 1D interacting electron due to potential barriers. Their results triggered the study of the problem of local perturbations to Luttinger liquid and Kondo problem in Luttinger liquid. The Kondo problem in Luttinger liquid was considered by Lee and Toner [6]. They also performed the renormalization group calculation and found the crossover of the Kondo temperature from power law dependence on the Kondo coupling constant to an exponential one. Relying on poor man's scaling method, Frusaki and Nagaosa [7] showed that the Kondo coupling flows to the strong-coupling regime not only for the antiferromagnetic case but also for the ferromagnetic case. The boundary conformal field theory [8] allows a classification of critical behavior for Luttinger liquid coupled to a magnetic impurity. It turns out that there are two possibilities, a local Fermi liquid with standard low-temperature thermodynamics or a non-Fermi liquid [7]. The non-Fermi liquid behavior is induced by the tunneling effect of conduction electrons through the impurity which depends only on the bulk properties but not on the details of the impurity [9]. Density matrix renormalization group calculation also supports the same conclusion [10]. In addition the renormalization group flow diagram for parameters characterizing impurity is more complex and contains fixed points responsible for the low temperature behaviors when the potential of impurity is not strong [11].

Despite all important progress hitherto made, the problem of few impurities embedded in a strongly correlated 1D electron system is still far from a complete understanding. We think that exact solutions of some integrable models on the subjects are useful from which one can expect to draw definite conclusions. Indeed Bedüfig *et al* has thoroughly solved an integrable model with impurity coupled with $t - J$ chain [12]. They introduce the impurity through a local vertices as in [13]. The model introduced suffers the lack of backward scattering and the presence of redundant terms in the hamiltonian. Based on Kane and Fisher's observation [5], we see it is advantageous to use open boundary problem with the impurities at open ends to study the problem of impurities coupled with strongly-correlated electron system. The program has been initiated for δ -interacting fermi system in [14] for $t - J$ model in [15] and for Hubbard model in [16].

The $t - J$ model, is considered as one of the most fundamental models in strongly correlated electron system for its possible relevance for purely electronic mechanisms for high- T_c superconductivity and heavy-fermion system. This model is obtained from the Hubbard model as an effective hamiltonian for the low-energy states in the strong-correlation limit. In this limit double occupancy of fermions is forbidden, leading to only three possible states at each lattice site for half spin. Currently, there is upsurge for its study. Very recently, the Luttinger liquid properties of the $t - J$ model are discussed in Ref. [17]. By solving the functional relations, the finite-size corrections related to $t - J$ model are dealt with for the open boundary conditions in Ref. [18]. The effects about an integrable impurity coupling to both spin and charge degrees of freedom are studied in a periodic $t - J$ chain [12] which we have mentioned above. The another generalization of the $t - J$ model is given in Ref. [19] by using the one-parametric family of four-dimensional representations of $gl(2|1)$. It is also a kind of generalization of extended Hubbard model [20].

In this paper we expand the study of the Kondo problem in 1D $t - J$ model [15] by exact solution

of open boundary Bethe ansatz. For this purpose we put two magnetic impurities in both sides of the open $t - J$ model which is a typical situation for the one-dimensional systems with impurities. The coupling constants of the impurities with conduction electrons cover from negative infinity to positive infinity, which means that both the ferromagnetic Kondo effect and antiferromagnetic Kondo effect can be dealt with on the same setting. We then construct the hamiltonian for the system with magnetic impurities from $t - J$ model. The integrability of this model ensures that both the Yang-Baxter equation and the reflecting Yang-Baxter equation are satisfied. By using the algebraic Bethe ansatz scheme for open boundary [21] we diagonalize the hamiltonian for the present system and obtain the Bethe ansatz equations. From which we derive the nonlinear integral equations governing the thermodynamic properties of the model for large system. The finite-size corrections for energy of ground-states in all cases can be calculated.

The arrangement of the present article is as follows. In section 2 the constructed hamiltonian and its first quantization form are given explicitly. In section 3 the boundary matrix depending on the rapidity and spin of the particle is given and all possible integrable cases for the model are exhausted. The Bethe ansatz equations of the systems for all integrable cases are derived in section 4. The properties of the ground state for the cases other than that in [15] are discussed in great detail in section 5. In the final section the finite-size corrections of the ground-state energies for chosen cases are obtained.

II. THE HAMILTONIAN OF THE MODEL

Consider one-dimensional lattice with G sites, N electrons and two magnetic impurities at both ends. Due to a large on-site Coulomb repulsion there are at most one particle at one site. The dynamics of the system governed by a hamiltonian which we construct from the $t - J$ model [22-27]. The conduction electrons can hop (t) between the neighbor sites. There are four types interactions in the model. A spin exchange interaction J and a charge interaction independent of the spin of strength V between the neighbor conduction electron; Kondo coupling J_a, J_b and impurity potential interactions V_a, V_b between the electron and impurities. The hamiltonian of the system has the form:

$$H = -t \sum_{j=1}^{G-1} \sum_{\sigma=\uparrow\downarrow} (C_{j\sigma}^+ C_{j+1\sigma} + C_{j+1\sigma}^+ C_{j\sigma}) + J \sum_{j=1}^{G-1} \mathbf{S}_j \cdot \mathbf{S}_{j+1} + V \sum_{j=1}^{G-1} n_j n_{j+1} + J_a \mathbf{S}_1 \cdot \mathbf{S}_a + V_a n_1 + J_b \mathbf{S}_G \cdot \mathbf{S}_b + V_b n_G, \quad (1)$$

where $C_{j\sigma}^+ (C_{j\sigma})$ is the creation (annihilation) operator of the conduction electron with spin σ on the site j ; $J_{a,b}, V_{a,b}$ are the Kondo coupling constants and the impurity potentials, respectively; $\mathbf{S}_j = \frac{1}{2} \sum_{\sigma,\sigma'} C_{j\sigma}^+ \sigma_{\sigma,\sigma'} C_{j\sigma'}$ is the spin operator of the conduction electron; $n_j = C_{j\uparrow}^+ C_{j\uparrow} + C_{j\downarrow}^+ C_{j\downarrow}$ is the number operator of the conduction electron; G is the length (or site number) of the system. Some properties of the ground state for $t = 1, J = 2, V = 3/2$ have been reported in Ref. [15]. Following Schultz's notation [28] we write the translation operators T_j^\pm :

$$T_j^\pm \Psi(x_1, \dots, x_j, \dots, x_N) = \Psi(x_1, \dots, x_j \pm 1, \dots, x_N),$$

where $\Psi(x_1, \dots, x_j, \dots, x_N)$ is the wave function of N conduction electrons. In first quantization and in appropriate energy units ($t = 1$) the hamiltonian of this system can be written down as

$$H = - \sum_{j=1}^N (T_j^+ + T_j^-) + \sum_{j=1}^N (K_{aj} \delta_{x_j,1} + K_{bj} \delta_{x_j,G} + K_j) \quad (2)$$

where the couplings are denoted by operators $K_{aj} = V_a - \frac{J_a}{4} + \frac{J_a}{2} P_{aj}$ and $K_{bj} = V_b - \frac{J_b}{4} + \frac{J_b}{2} P_{bj}$ with the permutation operators $P_{a(b),j}$ between the spins of the conduction electron j and the impurities $a, (b)$. The operator K_j acts on the wave function Ψ as

$$K_j \Psi(x_1, x_2, \dots, x_N) = \sum_{i=1}^N \delta_{x_j, x_i+1} K_{ij} \Psi(x_1, x_2, \dots, x_N),$$

where $K_{ij} = V - \frac{J}{4} + \frac{J}{2} P_{ij}$ describes the interactions between the conduction electrons with the permutation operator P_{ij} permuting i -th and j -th electron in spin space. We will diagonalize the above hamiltonian in the following section.

III. INTEGRABILITY CONDITIONS

We write the wave function in region $0 \leq x_{Q1} \leq x_{Q2} \leq \dots \leq x_{QN} \leq L-1$ as

$$\begin{aligned} & \Psi_{\sigma_1, \sigma_2, \dots, \sigma_N}(x_1, x_2, \dots, x_N) \\ &= \sum_P \sum_{r_1, r_2, \dots, r_N = \pm 1} \varepsilon_P \varepsilon_r A_{\sigma_{Q1}, \sigma_{Q2}, \dots, \sigma_{QN}}(r_{PQ1} k_{PQ1}, r_{PQ2} k_{PQ2}, \dots, r_{PQN} k_{PQN}) \\ & \cdot \exp[i \sum_{j=1}^N r_{Pj} k_{Pj} x_j] \end{aligned} \quad (3)$$

where the coefficients $A_{\sigma_{Q1}, \sigma_{Q2}, \dots, \sigma_{QN}}(r_{PQ1} k_{PQ1}, r_{PQ2} k_{PQ2}, \dots, r_{PQN} k_{PQN})$ are also dependent on the spins of magnetic impurities which are suppressed for brevity, and $\varepsilon_P = 1(-1)$, when the parity of P is even(odd), $\varepsilon_r = \prod_{j=1}^N r$ in which r takes the value $+1$ or -1 . The boundary R matrix satisfies the reflecting Yang-Baxter equation:

$$S_{12}(\lambda, \mu) \overset{1}{R}(\lambda) S_{12}(\lambda, -\mu) \overset{2}{R}(\mu) = \overset{2}{R}(\mu) S_{12}(\lambda, -\mu) \overset{1}{R}(\lambda) S_{12}(\lambda, \mu), \quad (4)$$

where operators $\overset{1}{R}(\lambda)$ and $\overset{2}{R}(\mu)$ are defined as

$$\overset{1}{R}(\lambda) = R(\lambda) \otimes id_{V_2}, \quad \overset{2}{R}(\mu) = id_{V_1} \otimes R(\mu)$$

for matrix $R \in \text{End}(V)$. S matrix satisfies the normal factorizable condition:

$$S_{12}(k, \lambda) S_{13}(k, \mu) S_{23}(\lambda, \mu) = S_{23}(\lambda, \mu) S_{13}(k, \mu) S_{12}(k, \lambda). \quad (5)$$

For convenience we set $t = 1$. From reflecting Yang-Baxter equation and the form for S matrix, we refer that the boundary R matrix should have the form

$$R = \exp(i\varphi) \frac{q - iC - iP}{q + iC + iP}, \quad (6)$$

where P is the permutation operator, $q = \pm \frac{1}{2} \cot \frac{k}{2}$, $\pm \frac{1}{2} \tan \frac{k}{2}$ and C is the arbitrary constant. Putting $K_{a(b),j} = m + lP$, we have from eq. (4) that

$$q \left[(m-1)^2 - l^2 \right] \tan^2 \frac{k}{2} + 2l (q^2 + C^2 - 1) \tan \frac{k}{2} + q \left[(m+1)^2 - l^2 \right] = 0. \quad (7)$$

This is the restriction imposed on coupling constants in order that our model (1) to be integrable. The details are as follows.

$$J = 2, \quad V = -\frac{1}{2}$$

In this case we know that the scattering matrix in the bulk can be written as:

$$S_{12}(k_1, k_2) = \frac{\frac{1}{2} \cot \frac{k_1}{2} - \frac{1}{2} \cot \frac{k_2}{2} - iP_{12}}{\frac{1}{2} \cot \frac{k_1}{2} - \frac{1}{2} \cot \frac{k_2}{2} - i} \quad (8)$$

where P_{12} is the permutation operator between two electrons. The boundary R matrix at the left end of the chain takes the form:

$$R_a(k_j, \sigma_j) = \exp[i\varphi_a(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - iP_{aj}}{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + iP_{aj}}. \quad (9)$$

The coupling constants J_a , V_a at the left end of the chain are expressed in terms, of C_a

$$J_a = -\frac{8}{(2C_a \mp 1)(2C_a \pm 3)}, \quad V_a = \frac{3 - 4C_a^2}{(2C_a \mp 1)(2C_a \pm 3)}, \quad (10)$$

and

$$\exp[i\varphi_a(k_j)] = \frac{J_a(\cot \frac{k_j}{2} + 2iC_a) \exp(ik_j) + i[4 + (4V_a - J_a) \exp(ik_j)]}{J_a(\cot \frac{k_j}{2} - 2iC_a) \exp(-ik_j) - i[4 + (4V_a - J_a) \exp(-ik_j)]}. \quad (11)$$

The boundary R matrix at the right end of the chain has the form:

$$R_b(-k_j, \sigma_j) = \exp[-2ik_j(G+1) + i\varphi_b(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_b - iP_{bj}}{\frac{1}{2} \cot \frac{k_j}{2} + iC_b + iP_{bj}}. \quad (12)$$

Similar relations exist for J_b , V_b and $\varphi_b(k_j)$ by merely substituting indices a in (10) and (11) by b .

$$\mathbf{A.} \quad J = -2 \quad V = \frac{1}{2}$$

The boundary R matrices have the forms:

$$R_a(k_j, \sigma_j) = \exp[i\varphi_a(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + iP_{aj}}{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - iP_{aj}}, \quad (13)$$

$$R_b(-k_j, \sigma_j) = \exp[-2ik_j(G+1) + i\varphi_b(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + iP_{bj}}{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - iP_{bj}}. \quad (14)$$

$\varphi_a(k_j)$ and $\varphi_b(k_j)$ are the same as in the proceeding. Now the coupling constants should be written in terms of the arbitrary parameter C_a in the form

$$J_a = \frac{8}{(2C_a \mp 1)(2C_a \pm 3)}, \quad V_a = \frac{4C_a^2 - 3}{(2C_a \mp 1)(2C_a \pm 3)} \quad (15)$$

J_b , V_b have the same expressions except with the substitution of indices a by b . Correspondingly, the scattering matrix S in the bulk for two conduction electrons is

$$S_{12}(k_1, k_2) = \frac{\frac{1}{2} \tan \frac{k_1}{2} - \frac{1}{2} \tan \frac{k_2}{2} + iP_{12}}{\frac{1}{2} \tan \frac{k_1}{2} - \frac{1}{2} \tan \frac{k_2}{2} + i}. \quad (16)$$

$$\mathbf{B.} \quad J = 2 \quad V = \frac{3}{2}$$

In this case the dependence of coupling constants on parameter C_a takes the form

$$J_a = -\frac{8}{(2C_a \mp 1)(2C_a \pm 3)}, \quad V_a = \frac{4C_a^2 - 7}{(2C_a \mp 1)(2C_a \pm 3)}. \quad (17)$$

J_b , V_b have the same expressions by the substituting of indices a by b . The scattering matrix in the bulk is

$$S_{12}(k_1, k_2) = -\frac{\frac{1}{2} \tan \frac{k_1}{2} - \frac{1}{2} \tan \frac{k_2}{2} - iP_{12}}{\frac{1}{2} \tan \frac{k_1}{2} - \frac{1}{2} \tan \frac{k_2}{2} + i}. \quad (18)$$

The boundary R matrices are

$$R_a(k_j, \sigma_j) = \exp[i\varphi_a(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - iP_{aj}}{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + iP_{aj}}, \quad (19)$$

$$R_b(-k_j, \sigma_j) = \exp[-2ik_j(G+1) + i\varphi_b(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - iP_{bj}}{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + iP_{bj}}. \quad (20)$$

$$\mathbf{C.} \quad J = -2 \quad V = -\frac{3}{2}$$

The coupling constants have the forms

$$J_a = \frac{8}{(2C_a \mp 1)(2C_a \pm 3)}, \quad V_a = \frac{7 - 4C_a^2}{(2C_a \mp 1)(2C_a \pm 3)}. \quad (21)$$

J_b , V_b have the same expressions by the substituting of indices a by b . The scattering matrix in the bulk is

$$S_{12}(k_1, k_2) = -\frac{\frac{1}{2} \cot \frac{k_1}{2} - \frac{1}{2} \cot \frac{k_2}{2} + iP_{12}}{\frac{1}{2} \cot \frac{k_1}{2} - \frac{1}{2} \cot \frac{k_2}{2} - i}. \quad (22)$$

The boundary R matrices are

$$R_a(k_j, \sigma_j) = \exp[i\varphi_a(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + iP_{aj}}{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - iP_{aj}}, \quad (23)$$

$$R_b(-k_j, \sigma_j) = \exp[-2ik_j(G+1) + i\varphi_b(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} + iC_b + iP_{bj}}{\frac{1}{2} \cot \frac{k_j}{2} - iC_b - iP_{bj}}, \quad (24)$$

The expressions for boundary matrices depending on both the moment of the particle and the spin of the electron are new. The expressions of S matrix in the bulk have been obtained before in Ref. [22], but they are different from ours.

IV. BETHE ANSATZ EQUATIONS

By using the standard Bethe ansatz procedure, we can diagonalize the hamiltonian (1) [21] and obtain the following Bethe Ansatz equations. When $J = 2$ and $V = -\frac{1}{2}$, setting

$$S_{j0}(k_j, k_0) = \frac{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_0}{2} - iP_{j0}}{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_0}{2} - i}, \quad (25)$$

$$S_{jN+1}(k_j, k_{N+1}) = \frac{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_{N+1}}{2} - iP_{jN+1}}{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_{N+1}}{2} - i}, \quad (26)$$

where $\cot \frac{k_0}{2} = 2iC_a$, $\cot \frac{k_{N+1}}{2} = 2iC_b$, $P_{j0} \equiv P_{aj}$, $P_{jN+1} \equiv P_{bj}$, we can write down the boundary R matrices as the forms:

$$R_a(k_j, \sigma_j) = \exp[i\varphi_a(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + i} \frac{S_{j0}(k_j, k_0)}{S_{j0}(-k_j, k_0)}, \quad (27)$$

$$R_b(-k_j, \sigma_j) = \exp[-2ik_j(G+1) + i\varphi_b(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_b - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_b + i} \frac{S_{jN+1}(k_j, k_{N+1})}{S_{jN+1}(-k_j, k_{N+1})}. \quad (28)$$

Define

$$T(\lambda) = S_{\tau j}(\lambda) S_{\tau 0}(\lambda) S_{\tau 1}(\lambda) \cdots S_{\tau j-1}(\lambda) S_{\tau j+1}(\lambda) \cdots S_{\tau N}(\lambda) S_{\tau N+1}(\lambda) \quad (29)$$

with

$$S_{\tau l}(\lambda) = \frac{\lambda - \frac{1}{2} \cot \frac{k_l}{2} - iP_{\tau l}}{\lambda - \frac{1}{2} \cot \frac{k_l}{2} - i}, \quad l = 0, 1, \dots, N+1. \quad (30)$$

We get the equation

$$\begin{aligned} Tr [T(\lambda) T^{-1}(-\lambda)] \Big|_{\lambda = \frac{1}{2} \cot \frac{k_j}{2}} \Phi \\ = \frac{2i - \cot \frac{k_j}{2} \frac{1}{2} \cot \frac{k_j}{2} + iC_a + i \frac{1}{2} \cot \frac{k_j}{2} + iC_b + i}{i - \cot \frac{k_j}{2} \frac{1}{2} \cot \frac{k_j}{2} - iC_a - i \frac{1}{2} \cot \frac{k_j}{2} - iC_b - i} \\ \cdot \exp[-i\varphi_a(k_j) - i\varphi_b(k_j) + 2ik_j(G+1)] \Phi \end{aligned} \quad (31)$$

where Φ is the eigenstate of the system. Then the Bethe ansatz equations can be expressed as

$$\begin{aligned} \exp[2ik_j(G+1) - i\varphi_a(k_j) - i\varphi_b(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + i \frac{1}{2} \cot \frac{k_j}{2} + iC_b + i}{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - i \frac{1}{2} \cot \frac{k_j}{2} - iC_b - i} \\ = \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2}} \frac{\frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (j = 1, 2, \dots, N) \end{aligned} \quad (32)$$

$$\frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2 (\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 (\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}$$

$$= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M). \quad (33)$$

M is the number of down spins and N is the number of the electrons. The function φ is denoted by expression (11). Similarly, when $J = -2$ and $V = \frac{1}{2}$, we can write down the Bethe Ansatz equations as the forms:

$$\begin{aligned} & \exp[2ik_j(G+1) - i\varphi_a(k_j) - i\varphi_b(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - i}{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + i} \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - i}{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + i} \\ & = \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2}} \frac{\frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (j = 1, 2, \dots, N) \end{aligned} \quad (34)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2} \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\ & = \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M). \end{aligned} \quad (35)$$

When $J = 2$ and $V = \frac{3}{2}$, we have that

$$\begin{aligned} & \exp[2ik_j(G+1) - i\varphi_a(k_j) - i\varphi_b(k_j)] \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + i}{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - i} \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + i}{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - i} \\ & \cdot \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} + i}{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} - i} \frac{\frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} + i}{\frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} - i} \\ & = \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2}} \frac{\frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \end{aligned} \quad (36)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2} \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\ & = \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M). \end{aligned} \quad (37)$$

When $J = -2$ and $V = -\frac{3}{2}$, we get that

$$\begin{aligned} & \exp[2ik_j(G+1) - i\varphi_a(k_j) - i\varphi_b(k_j)] \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + i} \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_b - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_b + i} \\ & \cdot \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} - i}{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} + i} \frac{\frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} - i}{\frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} + i} \end{aligned}$$

$$= \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \quad (38)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2 (\lambda_\alpha + \frac{i}{2})^2 + C_b^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 (\lambda_\alpha - \frac{i}{2})^2 + C_b^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 (\lambda_\alpha - \frac{i}{2})^2 + C_b^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 (\lambda_\alpha - \frac{i}{2})^2 + C_b^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M). \end{aligned} \quad (39)$$

Here the function $\varphi_a(k_j)$ is expressed by equation (11) and $\varphi_b(k_j)$ has the same expression as relation (11) with the substitution of index a by b . M is the number of down spins and N is the number of the electrons. It should be noted that in the above Bethe Ansatz equations we have choose the boundary R matrices as the form as the ones in Section 3, where the boundary matrices depend on the spin parameter. If the R matrix is dependent on the spin of the electron only at one end of the chain, for example, we denote by $R_b(k_j, \hat{\sigma}_j)$ the boundary matrix at right end of the chain independent on the spin σ_j . The Bethe Ansatz equations for $J = 2$, $V = -\frac{1}{2}$ take the form:

$$\frac{\exp[-i\varphi_a(k_j)] \frac{1}{2} \cot \frac{k_j}{2} + iC_a + i}{R_b(-k_j, \hat{\sigma}_j) \frac{1}{2} \cot \frac{k_j}{2} - iC_a - i} = \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (40)$$

$$(j = 1, 2, \dots, N)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i}, \quad (\alpha = 1, 2, \dots, M) \end{aligned} \quad (41)$$

Similarly, when the boundary matrix at the left end of the spin is independent on the spin of the electron, denoted by $R_a(k_j, \hat{\sigma}_j)$, we have that

$$\begin{aligned} & \frac{\exp[2ik_j(G+1) - i\varphi_b(k_j)] \frac{1}{2} \cot \frac{k_j}{2} + iC_b + i}{R_a(k_j, \hat{\sigma}_j) \frac{1}{2} \cot \frac{k_j}{2} - iC_b - i} \\ &= \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (j = 1, 2, \dots, N) \end{aligned} \quad (42)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i}, \quad (\alpha = 1, 2, \dots, M) \end{aligned} \quad (43)$$

where the number of down spins should less than $N + 2$ and N is the number of the conduction electrons in the system. Furthermore, we get that

$$\frac{\exp[-i\varphi_a(k_j)]}{R_b(-k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - i}{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + i} = \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (44)$$

$$(j = 1, 2, \dots, N)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M) \end{aligned} \quad (45)$$

and

$$\begin{aligned} & \frac{\exp[2ik_j(G+1) - i\varphi_b(k_j)]}{R_a(k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - i}{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + i} \\ &= \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (j = 1, 2, \dots, N) \end{aligned} \quad (46)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M) \end{aligned} \quad (47)$$

for the case of $J = -2$, $V = \frac{1}{2}$.

$$\begin{aligned} & \frac{\exp[-i\varphi_a(k_j)]}{R_b(-k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_a + i}{\frac{1}{2} \tan \frac{k_j}{2} - iC_a - i} \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} + i \frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} + i}{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} - i \frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} - i} \\ &= \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \end{aligned} \quad (48)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M) \end{aligned} \quad (49)$$

and

$$\begin{aligned}
& \frac{\exp[2ik_j(G+1) - i\varphi_b(k_j)]}{R_a(k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \tan \frac{k_j}{2} + iC_b + i}{\frac{1}{2} \tan \frac{k_j}{2} - iC_b - i} \\
& \cdot \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} + i \frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} + i}{\frac{1}{2} \tan \frac{k_j}{2} - \frac{1}{2} \tan \frac{k_l}{2} - i \frac{1}{2} \tan \frac{k_j}{2} + \frac{1}{2} \tan \frac{k_l}{2} - i} \\
& = \prod_{\beta=1}^M \frac{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}{\frac{1}{2} \tan \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \tan \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \quad (50)
\end{aligned}$$

$$\begin{aligned}
& \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \tan \frac{k_l}{2} - \frac{i}{2}} \\
& = \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M) \quad (51)
\end{aligned}$$

for the case of $J = 2$, $V = \frac{3}{2}$ when boundary matrix only at one end of the chain rely on the spin parameter of the electron. Finally, for the case of $J = -2$ and $V = -\frac{3}{2}$, the Bethe Ansatz equations take the form:

$$\begin{aligned}
& \frac{\exp[-i\varphi_a(k_j)]}{R_b(-k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_a - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_a + i} \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} - i \frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} - i}{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} + i \frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} + i} \\
& = \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \quad (52)
\end{aligned}$$

$$\begin{aligned}
& \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2} \lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}} \\
& = \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M) \quad (53)
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\exp[2ik_j(G+1) - i\varphi_b(k_j)]}{R_a(k_j, \hat{\sigma}_j)} \frac{\frac{1}{2} \cot \frac{k_j}{2} - iC_b - i}{\frac{1}{2} \cot \frac{k_j}{2} + iC_b + i} \\
& \cdot \prod_{l=1(l \neq j)}^N \frac{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} - i \frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} - i}{\frac{1}{2} \cot \frac{k_j}{2} - \frac{1}{2} \cot \frac{k_l}{2} + i \frac{1}{2} \cot \frac{k_j}{2} + \frac{1}{2} \cot \frac{k_l}{2} + i} \\
& = \prod_{\beta=1}^M \frac{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta - \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta - \frac{i}{2}}{\frac{1}{2} \cot \frac{k_j}{2} - \lambda_\beta + \frac{i}{2} \frac{1}{2} \cot \frac{k_j}{2} + \lambda_\beta + \frac{i}{2}}, \quad (j = 1, 2, \dots, N), \quad (54)
\end{aligned}$$

$$\begin{aligned}
& \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha - \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}} \frac{\lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} + \frac{i}{2}}{\lambda_\alpha + \frac{1}{2} \cot \frac{k_l}{2} - \frac{i}{2}} \\
&= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \frac{\lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha + \lambda_\beta - i} \quad (\alpha = 1, 2, \dots, M). \tag{55}
\end{aligned}$$

$R_a(k_j, \hat{\sigma}_j)$ and $R_b(-k_j, \hat{\sigma}_j)$ denote that the boundary matrices at the left and the right ends of the system are independent on the spin σ_j respectively. Notice that the number of down spins is less than $N+2$ for the system with N conduction electrons. In the following section, we focus the discussions on the system with the boundary matrices depending on the spins of the electrons at both ends of the chain. Set

$$\begin{aligned}
\theta_a(k) &= \frac{1}{i} \ln \frac{(4C_a^2 - 3) \cos k - 4C_a^2 + 5 + 4iC_a \sin k}{(4C_a^2 - 3) \cos k - 4C_a^2 + 5 - 4iC_a \sin k}, \\
\theta_b(k) &= \frac{1}{i} \ln \frac{(4C_b^2 - 3) \cos k - 4C_b^2 + 5 + 4iC_b \sin k}{(4C_b^2 - 3) \cos k - 4C_b^2 + 5 - 4iC_b \sin k}. \tag{56}
\end{aligned}$$

From relation (11) and

$$\exp[i\varphi_b(k_j)] = \frac{J_b(\cot \frac{k_j}{2} + 2iC_b) \exp(ik_j) + i[4 + (4V_b - J_b) \exp(ik_j)]}{J_b(\cot \frac{k_j}{2} - 2iC_b) \exp(-ik_j) - i[4 + (4V_b - J_b) \exp(-ik_j)]} \tag{57}$$

we get the following expressions. When $J = 2$ and $V = -\frac{1}{2}$, we have that

$$\exp[i\varphi_a(k)] = \begin{cases} \exp(ik) & \text{for } J_a = -\frac{8}{(2C_a-1)(2C_a+3)}, V_a = \frac{3-4C_a^2}{(2C_a-1)(2C_a+3)} \\ \exp[ik + i\theta_a(k)] & \text{for } J_a = -\frac{8}{(2C_a+1)(2C_a-3)}, V_a = \frac{3-4C_a^2}{(2C_a+1)(2C_a-3)} \end{cases}. \tag{58}$$

When $J = -2$ and $V = \frac{1}{2}$, we have that

$$\exp[i\varphi_a(k)] = \begin{cases} -\exp(ik) & \text{for } J_a = \frac{8}{(2C_a-1)(2C_a+3)}, V_a = \frac{4C_a^2-3}{(2C_a-1)(2C_a+3)} \\ -\exp[ik - i\theta_a(\pi - k)] & \text{for } J_a = \frac{8}{(2C_a+1)(2C_a-3)}, V_a = \frac{4C_a^2-3}{(2C_a+1)(2C_a-3)} \end{cases}. \tag{59}$$

When $J = 2$ and $V = \frac{3}{2}$, we have that

$$\exp[i\varphi_a(k)] = \begin{cases} -\exp[ik + i\theta_a(\pi - k)] & \text{for } J_a = \frac{-8}{(2C_a-1)(2C_a+3)}, V_a = \frac{4C_a^2-7}{(2C_a-1)(2C_a+3)} \\ -\exp(ik) & \text{for } J_a = \frac{-8}{(2C_a+1)(2C_a-3)}, V_a = \frac{4C_a^2-7}{(2C_a+1)(2C_a-3)} \end{cases}. \tag{60}$$

When $J = -2$ and $V = -\frac{3}{2}$, we have that

$$\exp[i\varphi_a(k)] = \begin{cases} \exp[ik + i\theta_a(-k)] & \text{for } J_a = \frac{8}{(2C_a-1)(2C_a+3)}, V_a = \frac{7-4C_a^2}{(2C_a-1)(2C_a+3)} \\ \exp(ik) & \text{for } J_a = \frac{8}{(2C_a+1)(2C_a-3)}, V_a = \frac{7-4C_a^2}{(2C_a+1)(2C_a-3)} \end{cases}. \tag{61}$$

The expressions of $\exp[i\varphi_b(k)]$ can be obtained by substituting the index a of b in the above relations. Then, without loss any generalization, we can choose that

$$\begin{aligned}
J_a &= -\frac{8}{(2C_a-1)(2C_a+3)}, V_a = \frac{3-4C_a^2}{(2C_a-1)(2C_a+3)}, \\
J_b &= -\frac{8}{(2C_b-1)(2C_b+3)}, V_b = \frac{3-4C_b^2}{(2C_b-1)(2C_b+3)} \tag{62}
\end{aligned}$$

for $J = 2$ and $V = -\frac{1}{2}$. The Bethe Ansatz equations take the forms as

$$\frac{q_{j+i(C_a+1)} q_{j+i(C_b+1)}}{q_{j-i(C_a+1)} q_{j-i(C_b+1)}} \exp(2ik_j G) = \prod_{\beta=1}^M \frac{q_j - \lambda_\beta + \frac{i}{2} q_j + \lambda_\beta + \frac{i}{2}}{q_j - \lambda_\beta - \frac{i}{2} q_j + \lambda_\beta - \frac{i}{2}}, \quad (63)$$

$$\begin{aligned} & \frac{(\lambda_\alpha + \frac{i}{2})^2 + C_a^2 (\lambda_\alpha + \frac{i}{2})^2 + C_b^2}{(\lambda_\alpha - \frac{i}{2})^2 + C_a^2 (\lambda_\alpha - \frac{i}{2})^2 + C_b^2} \prod_{l=1}^N \frac{\lambda_\alpha - q_l + \frac{i}{2} \lambda_\alpha + q_l + \frac{i}{2}}{\lambda_\alpha - q_l - \frac{i}{2} \lambda_\alpha + q_l - \frac{i}{2}} \\ &= \prod_{\beta=1(\beta \neq \alpha)}^M \frac{\lambda_\alpha - \lambda_\beta + i \lambda_\alpha + \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i \lambda_\alpha + \lambda_\beta - i}, \end{aligned} \quad (64)$$

where $q_j = \frac{1}{2} \cot \frac{k_j}{2}$. For the case of $J = -2$ and $V = \frac{1}{2}$, the Bethe Ansatz equations have also the forms (63) and (64) with $q_j = -\frac{1}{2} \tan \frac{k_j}{2}$ and the Kondo coupling constants should be

$$\begin{aligned} J_a &= \frac{8}{(2C_a - 1)(2C_a + 3)}, V_a = \frac{4C_a^2 - 3}{(2C_a - 1)(2C_a + 3)}, \\ J_b &= \frac{8}{(2C_b - 1)(2C_b + 3)}, V_b = \frac{4C_b^2 - 3}{(2C_b - 1)(2C_b + 3)}. \end{aligned} \quad (65)$$

If $J = 2$ and $V = \frac{3}{2}$, we choose that

$$\begin{aligned} J_a &= -\frac{8}{(2C_a + 1)(2C_a - 3)}, V_a = \frac{4C_a^2 - 7}{(2C_a + 1)(2C_a - 3)}, \\ J_b &= -\frac{8}{(2C_b + 1)(2C_b - 3)}, V_b = \frac{4C_b^2 - 7}{(2C_b + 1)(2C_b - 3)}, \end{aligned} \quad (66)$$

and the Bethe Ansatz equations are

$$\begin{aligned} & \exp(2ik_j G) \frac{q_{j+i(C_a+1)} q_{j+i(C_b+1)}}{q_{j-i(C_a+1)} q_{j-i(C_b+1)}} \prod_{l=1(l \neq j)}^N \frac{q_j - q_l + i q_j + q_l + i}{q_j - q_l - i q_j + q_l - i} \\ &= \prod_{\beta=1}^M \frac{q_j - \lambda_\beta + \frac{i}{2} q_j + \lambda_\beta + \frac{i}{2}}{q_j - \lambda_\beta - \frac{i}{2} q_j + \lambda_\beta - \frac{i}{2}} \end{aligned} \quad (67)$$

and relation (64) with $q_j = \frac{1}{2} \tan \frac{k_j}{2}$. They are also the Bethe Ansatz equations for $J = -2$ and $V = -\frac{3}{2}$ with $q_j = -\frac{1}{2} \cot \frac{k_j}{2}$ and

$$\begin{aligned} J_a &= \frac{8}{(2C_a + 1)(2C_a - 3)}, V_a = \frac{7 - 4C_a^2}{(2C_a + 1)(2C_a - 3)}, \\ J_b &= \frac{8}{(2C_b + 1)(2C_b - 3)}, V_b = \frac{7 - 4C_b^2}{(2C_b + 1)(2C_b - 3)}. \end{aligned} \quad (68)$$

V. GROUND STATE

In this paper we restrict the discussions of the properties of ground state to the case of $J = \pm 2$ and $V = \mp \frac{1}{2}$. The case of $J = 2$ and $V = \frac{3}{2}$ were studied in [15]. The eigenvalue of the hamiltonian is

$$E = \mp 2N \pm \sum_{j=1}^N \frac{1}{q_j^2 + \frac{1}{4}} \quad (69)$$

for $J = 2$, $V = -\frac{1}{2}$ with $q_j = \frac{1}{2} \cot \frac{k_j}{2}$ and $J = -2$, $V = \frac{1}{2}$ with $q_j = -\frac{1}{2} \tan \frac{k_j}{2}$, respectively. They satisfy the Bethe Ansatz equations (63) and (64) from which the integral equations are derived.

A. Integral Equations

Following [29], we introduce the notation

$$e(x) \equiv \frac{x+i}{x-i}.$$

Then, from relations (63) and (64) we get

$$e\left(\frac{q_j}{1+C_a}\right) e\left(\frac{q_j}{1+C_b}\right) e(2q_j)^{2G} = \prod_{\beta=1}^M e(2q_j - 2\lambda_\beta) e(2q_j + 2\lambda_\beta), \quad (70)$$

$$e\left(\frac{\lambda_\alpha}{\frac{1}{2}-C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2}+C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2}-C_b}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2}+C_b}\right)$$

$$\cdot \prod_{l=1}^N e(2\lambda_\alpha - 2q_l) e(2\lambda_\alpha + 2q_l) = \prod_{\beta=1(\beta \neq \alpha)}^M e(\lambda_\alpha - \lambda_\beta) e(\lambda_\alpha + \lambda_\beta), \quad (71)$$

where $j = 1, 2, \dots, N$; $\alpha = 1, 2, \dots, M$ and $e(\pm\infty) = 1$. Considering that the parameter q_j can take complex values, the general structure for $\{q_j\}_{j=1,2,\dots,N}$ should be consisting of M' pairs of $q_\alpha^\pm = \lambda_\alpha \pm \frac{i}{2} + O(\exp(-\delta G))$, $\alpha = 1, \dots, M'$ and M'' pairs of $\tilde{q}_\alpha^\pm = -\tilde{\lambda}_\alpha \pm \frac{i}{2} + O(\exp(-\delta G))$, $\tilde{\lambda}_\alpha \in \{\lambda_\beta\}$ and remaining $N - 2(M' + M'')$ non-pairing q_j s. To be more precise, we use

$$Q \equiv \{q_j \mid j = 1, 2, \dots, N\} = X' \cup X'' \cup Y,$$

where

$$\begin{aligned} X' &= \{q_\alpha^\pm = \lambda_\alpha \pm \frac{i}{2} + O(\exp(-\delta G)) \mid \alpha = 1, \dots, M'\}, \\ X'' &= \{\tilde{q}_\alpha^\pm = -\tilde{\lambda}_\alpha \pm \frac{i}{2} + O(\exp(-\delta G)) \mid \tilde{\lambda}_\alpha \in \{\lambda_\beta\}, \alpha = 1, \dots, M''\}, \\ Y &= Q \setminus (X' \cup X''). \end{aligned} \quad (72)$$

Obviously, the non-pairing q_j satisfies equation (70) with $j = 1, 2, \dots, N - 2(M' + M'')$. When $q_j \in X'$, from equation (70), we have

$$e\left(\frac{\lambda_\alpha}{\frac{3}{2}+C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2}+C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{3}{2}+C_b}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2}+C_b}\right) e(\lambda_\alpha)^{2G}$$

$$= e(2q_\alpha^+ - 2\lambda_\alpha)e(2q_\alpha^- - 2\lambda_\alpha)e(2\lambda_\alpha) \prod_{\beta=1, (\beta \neq \alpha)}^M e(\lambda_\alpha - \lambda_\beta)e(\lambda_\alpha + \lambda_\beta), \alpha = 1, 2, \dots, M'. \quad (73)$$

From equation (71) we have

$$\begin{aligned} & e\left(\frac{\lambda_\alpha}{\frac{1}{2} - C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2} + C_a}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2} - C_b}\right) e\left(\frac{\lambda_\alpha}{\frac{1}{2} + C_b}\right) e(2\lambda_\alpha) \\ & \cdot \prod_{l=1}^{N-2M_-} e(2\lambda_\alpha - 2q_l)e(2\lambda_\alpha + 2q_l) \prod_{\beta=1}^{M-M_-} e(\hat{\lambda}_\beta - \lambda_\alpha)e(-\hat{\lambda}_\beta - \lambda_\alpha) \\ & = e(2q_\alpha^+ - 2\lambda_\alpha)e(2q_\alpha^- - 2\lambda_\alpha) \end{aligned} \quad (74)$$

where $M_- = M' + M''$ and $\hat{\lambda}_\beta$ ($\beta = 1, 2, \dots, M - M_-$) are the parameters describing the down spins but having no contributions to the bound states. With the help of the above relation, equation (73) becomes

$$\begin{aligned} & e\left(\frac{\lambda_\alpha}{C_a + \frac{3}{2}}\right) e\left(\frac{\lambda_\alpha}{C_a - \frac{1}{2}}\right) e\left(\frac{\lambda_\alpha}{C_b + \frac{3}{2}}\right) e\left(\frac{\lambda_\alpha}{C_b - \frac{1}{2}}\right) e(\lambda_\alpha)^{2G} \\ & = e(2\lambda_\alpha)^2 \prod_{l=1}^{N-2M_-} e(2\lambda_\alpha - 2q_l)e(2\lambda_\alpha + 2q_l) \prod_{\beta=1 (\beta \neq \alpha)}^{M'} e(\lambda_\alpha - \lambda_\beta)e(\lambda_\alpha + \lambda_\beta) \\ & \cdot \prod_{\beta=1}^{M''} e(\lambda_\alpha - \tilde{\lambda}_\beta)e(\lambda_\alpha + \tilde{\lambda}_\beta), \quad \alpha = 1, \dots, M'. \end{aligned} \quad (75)$$

Similarly, when $q_j \in X''$, we get the following equation

$$\begin{aligned} & e\left(\frac{\tilde{\lambda}_\alpha}{C_a + \frac{3}{2}}\right) e\left(\frac{\tilde{\lambda}_\alpha}{C_a - \frac{1}{2}}\right) e\left(\frac{\tilde{\lambda}_\alpha}{C_b + \frac{3}{2}}\right) e\left(\frac{\tilde{\lambda}_\alpha}{C_b - \frac{1}{2}}\right) e(\tilde{\lambda}_\alpha)^{2G} \\ & = e(2\tilde{\lambda}_\alpha)^2 \prod_{l=1}^{N-2M_-} e(2\tilde{\lambda}_\alpha - 2q_l)e(2\tilde{\lambda}_\alpha + 2q_l) \prod_{\beta=1}^{M'} e(\tilde{\lambda}_\alpha - \lambda_\beta)e(\tilde{\lambda}_\alpha + \lambda_\beta) \\ & \cdot \prod_{\beta=1 (\beta \neq \alpha)}^{M''} e(\tilde{\lambda}_\alpha - \tilde{\lambda}_\beta)e(\tilde{\lambda}_\alpha + \tilde{\lambda}_\beta), \quad \alpha = 1, \dots, M'' \end{aligned} \quad (76)$$

The two equations (ee11) and (ee12) can be combined in a single equation.

$$\begin{aligned} & e\left(\frac{\lambda_\alpha}{C_a + \frac{3}{2}}\right) e\left(\frac{\lambda_\alpha}{C_a - \frac{1}{2}}\right) e\left(\frac{\lambda_\alpha}{C_b + \frac{3}{2}}\right) e\left(\frac{\lambda_\alpha}{C_b - \frac{1}{2}}\right) e(\lambda_\alpha)^{2G} \\ & = e(2\lambda_\alpha)^2 \prod_{l=1}^{N-2M_-} e(2\lambda_\alpha - 2q_l)e(2\lambda_\alpha + 2q_l) \\ & \cdot \prod_{\beta=1 (\beta \neq \alpha)}^{M_-} e(\lambda_\alpha - \lambda_\beta)e(\lambda_\alpha + \lambda_\beta), \quad \alpha = 1, 2, \dots, M_-, \end{aligned} \quad (77)$$

with the new λ_α defined by

$$\lambda_\alpha = \begin{cases} \lambda_\alpha & \text{when } \alpha = 1, 2, \dots, M' \\ \tilde{\lambda}_{M'-\alpha} & \text{when } \alpha = M' + 1, M' + 2, \dots, M_- \end{cases}$$

The parameters $\hat{\lambda}_\alpha$ ($\alpha = 1, 2, \dots, M - M_-$), in view of (74), satisfy

$$\begin{aligned} & e\left(\frac{\hat{\lambda}_\alpha}{C_a + \frac{1}{2}}\right) e\left(\frac{\hat{\lambda}_\alpha}{C_b + \frac{1}{2}}\right) \prod_{l=1}^{N-2M_-} e(2\hat{\lambda}_\alpha - 2q_l) e(2\hat{\lambda}_\alpha + 2q_l) \\ &= e\left(\frac{\hat{\lambda}_\alpha}{C_a - \frac{1}{2}}\right) e\left(\frac{\hat{\lambda}_\alpha}{C_b - \frac{1}{2}}\right) \prod_{\beta=1(\beta \neq \alpha)}^{M-M_-} e(\hat{\lambda}_\alpha - \hat{\lambda}_\beta) e(\hat{\lambda}_\alpha + \hat{\lambda}_\beta). \end{aligned} \quad (78)$$

The non-pairing q_j (i.e. $q_j \in Y$) satisfies

$$\begin{aligned} e\left(\frac{q_j}{C_a + 1}\right) e\left(\frac{q_j}{C_b + 1}\right) e(2q_j)^{2G} &= \prod_{\beta=1}^{M_-} e(2q_j - 2\lambda_\beta) e(2q_j + 2\lambda_\beta) \\ &\quad \cdot \prod_{\beta=1}^{M-M_-} e(2q_j - 2\hat{\lambda}_\beta) e(2q_j + 2\hat{\lambda}_\beta), \end{aligned} \quad (79)$$

where $j = 1, 2, \dots, N - 2M_-$ and $e(\pm\infty) = 1$. Setting

$$\theta(x) \equiv 2 \tan^{-1} x, \quad -\pi < \theta \leq \pi,$$

we have

$$e(x) = \exp[i(\pi - \theta(x))].$$

The logarithms of the equations (77), (78) and (79) give, respectively,

$$\begin{aligned} & \theta\left(\frac{\lambda_\alpha}{C_a + \frac{3}{2}}\right) + \theta\left(\frac{\lambda_\alpha}{C_a - \frac{1}{2}}\right) + \theta\left(\frac{\lambda_\alpha}{C_b + \frac{3}{2}}\right) + \theta\left(\frac{\lambda_\alpha}{C_b - \frac{1}{2}}\right) + 2G\theta(\lambda_\alpha) \\ &= 4\pi J_\alpha + \theta(2\lambda_\alpha) + \sum_{l=1}^{N-2M_-} [\theta(2\lambda_\alpha - 2q_l) + \theta(2\lambda_\alpha + 2q_l)] \\ &\quad + \sum_{\beta=1}^{M_-} [\theta(\lambda_\alpha - \lambda_\beta) + \theta(\lambda_\alpha + \lambda_\beta)] \end{aligned} \quad (80)$$

with $\alpha = 1, 2, \dots, M_-$ and integers or half-integer J_α ;

$$\begin{aligned} & \theta\left(\frac{\hat{\lambda}_\alpha}{C_a + \frac{1}{2}}\right) + \theta\left(\frac{\hat{\lambda}_\alpha}{C_b + \frac{1}{2}}\right) + \sum_{l=1}^{N-2M_-} [\theta(2\hat{\lambda}_\alpha - 2q_l) + \theta(2\hat{\lambda}_\alpha + 2q_l)] \\ &= 4\pi \hat{J}_\alpha - \theta(2\lambda_\alpha) + \theta\left(\frac{\hat{\lambda}_\alpha}{C_a - \frac{1}{2}}\right) + \theta\left(\frac{\hat{\lambda}_\alpha}{C_b - \frac{1}{2}}\right) \\ &\quad + \sum_{\beta=1}^{M-M_-} [\theta(\hat{\lambda}_\alpha - \hat{\lambda}_\beta) + \theta(\hat{\lambda}_\alpha + \hat{\lambda}_\beta)] \end{aligned} \quad (81)$$

with $\alpha = 1, 2, \dots, M - M_-$ and integers or half-integer \widehat{J}_α ;

$$\begin{aligned} & \theta\left(\frac{q_j}{C_a+1}\right) + \theta\left(\frac{q_j}{C_b+1}\right) + 2G\theta(2q_j) \\ &= 4\pi I_j + \sum_{\beta=1}^{M_-} [\theta(2q_j - 2\lambda_\beta) + \theta(2q_j + 2\lambda_\beta)] + \sum_{\beta=1}^{M-M_-} [\theta(2q_j - 2\widehat{\lambda}_\beta) + \theta(2q_j + 2\widehat{\lambda}_\beta)] \end{aligned} \quad (82)$$

with $j = 1, 2, \dots, N - 2M_-$ and integers or half-integer I_j . By setting

$$\frac{d}{dx}\theta[k(x+c)] = 2\pi a\left(x+c, \frac{1}{k}\right), \quad (83)$$

the equations (80), (81) and (82) can be changed into the forms

$$\begin{aligned} & a\left(\lambda_\alpha, C_a + \frac{3}{2}\right) + a\left(\lambda_\alpha, C_a - \frac{1}{2}\right) + a\left(\lambda_\alpha, C_b + \frac{3}{2}\right) + a\left(\lambda_\alpha, C_b - \frac{1}{2}\right) + 2Ga(\lambda_\alpha, 1) \\ &= \frac{dJ_\alpha}{d\lambda_\alpha} + a\left(\lambda_\alpha, \frac{1}{2}\right) + \sum_{l=1}^{N-2M_-} \left[a\left(\lambda_\alpha - q_l, \frac{1}{2}\right) + a\left(\lambda_\alpha + q_l, \frac{1}{2}\right) \right] \\ & \quad + \sum_{\beta=1}^{M_-} [a(\lambda_\alpha - \lambda_\beta, 1) + a(\lambda_\alpha + \lambda_\beta, 1)] \end{aligned} \quad (84)$$

with $\alpha = 1, 2, \dots, M_-$;

$$\begin{aligned} & a\left(\widehat{\lambda}_\alpha, C_a + \frac{1}{2}\right) + a\left(\widehat{\lambda}_\alpha, C_b + \frac{1}{2}\right) + \sum_{l=1}^{N-2M_-} \left[a\left(\widehat{\lambda}_\alpha - q_l, \frac{1}{2}\right) + a\left(\widehat{\lambda}_\alpha + q_l, \frac{1}{2}\right) \right] \\ &= \frac{d\widehat{J}_\alpha}{d\widehat{\lambda}_\alpha} - a\left(\widehat{\lambda}_\alpha, \frac{1}{2}\right) + a\left(\widehat{\lambda}_\alpha, C_a - \frac{1}{2}\right) + a\left(\widehat{\lambda}_\alpha, C_b - \frac{1}{2}\right) \\ & \quad + \sum_{\beta=1}^{M-M_-} [a(\widehat{\lambda}_\alpha - \widehat{\lambda}_\beta, 1) + a(\widehat{\lambda}_\alpha + \widehat{\lambda}_\beta, 1)] \end{aligned} \quad (85)$$

with $\alpha = 1, 2, \dots, M - M_-$;

$$\begin{aligned} & a(q_j, C_a + 1) + a(q_j, C_b + 1) + 2Ga\left(q_j, \frac{1}{2}\right) \\ &= \frac{dI_j}{dq_j} + \sum_{\beta=1}^{M_-} \left[a\left(q_j - \lambda_\beta, \frac{1}{2}\right) + a\left(q_j + \lambda_\beta, \frac{1}{2}\right) \right] \\ & \quad + \sum_{\beta=1}^{M-M_-} \left[a\left(q_j - \widehat{\lambda}_\beta, \frac{1}{2}\right) + a\left(q_j + \widehat{\lambda}_\beta, \frac{1}{2}\right) \right] \end{aligned} \quad (86)$$

with $j = 1, 2, \dots, N - 2M_-$. We define that

$$j(\lambda) \equiv \theta(\lambda) +$$

$$\begin{aligned}
& + \frac{1}{2G} \left\{ \theta \left(\frac{\lambda}{C_a + \frac{3}{2}} \right) + \theta \left(\frac{\lambda}{C_a - \frac{1}{2}} \right) + \theta \left(\frac{\lambda}{C_b + \frac{3}{2}} \right) + \theta \left(\frac{\lambda}{C_b - \frac{1}{2}} \right) - \theta(2\lambda) \right\} \\
& - \frac{1}{2G} \left\{ \sum_{l=1}^{N-2M_-} [\theta(2\lambda - 2q_l) + \theta(2\lambda + 2q_l)] + \sum_{\beta=1}^{M_-} [\theta(\lambda - \lambda_\beta) + \theta(\lambda + \lambda_\beta)] \right\}, \tag{87}
\end{aligned}$$

$$\begin{aligned}
\hat{j}(\hat{\lambda}) & \equiv \frac{1}{2G} \left\{ \theta \left(\frac{\hat{\lambda}}{C_a + \frac{1}{2}} \right) - \theta \left(\frac{\hat{\lambda}}{C_a - \frac{1}{2}} \right) + \theta \left(\frac{\hat{\lambda}}{C_b + \frac{1}{2}} \right) - \theta \left(\frac{\hat{\lambda}}{C_b - \frac{1}{2}} \right) + \theta(2\hat{\lambda}) \right\} \\
& + \frac{1}{2G} \left\{ \sum_{l=1}^{N-2M_-} [\theta(2\hat{\lambda} - 2q_l) + \theta(2\hat{\lambda} + 2q_l)] - \sum_{\beta=1}^{M-M_-} [\theta(\hat{\lambda} - \hat{\lambda}_\beta) + \theta(\hat{\lambda} + \hat{\lambda}_\beta)] \right\}, \tag{88}
\end{aligned}$$

$$\begin{aligned}
h(q) & \equiv \theta(2q) + \frac{1}{2G} \left\{ \theta \left(\frac{q}{C_a + 1} \right) + \theta \left(\frac{q}{C_b + 1} \right) \right\} \\
& - \frac{1}{2G} \left\{ \sum_{\beta=1}^{M_-} [\theta(2q - 2\lambda_\beta) + \theta(2q + 2\lambda_\beta)] + \sum_{\beta=1}^{M-M_-} [\theta(2q - 2\hat{\lambda}_\beta) + \theta(2q + 2\hat{\lambda}_\beta)] \right\}. \tag{89}
\end{aligned}$$

Then, the holes of $\lambda, \hat{\lambda}$ and q are defined as the solutions of

$$\begin{aligned}
Gj(\lambda) & = 2\pi \times (\text{omitted } J), \\
G\hat{j}(\hat{\lambda}) & = 2\pi \times (\text{omitted } \hat{J}), \\
Gh(q) & = 2\pi \times (\text{omitted } I).
\end{aligned} \tag{90}$$

By taking the thermodynamic limits, we introduce the distribution functions

$$\left. \begin{aligned} \lambda & \rightarrow \sigma(\lambda) \\ q & \rightarrow \varrho(q) \\ \hat{\lambda} & \rightarrow \hat{\sigma}(\hat{\lambda}) \end{aligned} \right\} \xrightarrow{\text{holes}} \left\{ \begin{aligned} \sigma^h(\lambda) \\ \varrho^h(q) \\ \hat{\sigma}^h(\hat{\lambda}) \end{aligned} \right\}.$$

So we have that

$$\begin{aligned}
\frac{dj(\lambda)}{d\lambda} & = 2\pi (\sigma(\lambda) + \sigma^h(\lambda)), \\
\frac{dh(q)}{dq} & = 2\pi (\rho(q) + \varrho^h(q)), \\
\frac{d\hat{j}(\hat{\lambda})}{d\hat{\lambda}} & = 2\pi (\hat{\sigma}(\hat{\lambda}) + \hat{\sigma}^h(\hat{\lambda})).
\end{aligned} \tag{91}$$

Therefore, the integral equations can be written down as

$$2a(\lambda, 1) + \frac{1}{G} \left[a \left(\lambda, C_a + \frac{3}{2} \right) + a \left(\lambda, C_a - \frac{1}{2} \right) + a \left(\lambda, C_b + \frac{3}{2} \right) + a \left(\lambda, C_b - \frac{1}{2} \right) \right]$$

$$\begin{aligned}
&= \frac{1}{G} a\left(\lambda, \frac{1}{2}\right) + 2\sigma(\lambda) + 2\sigma^h(\lambda) + \int d\lambda' \sigma(\lambda') [a(\lambda - \lambda', 1) + a(\lambda + \lambda', 1)] \\
&\quad + \int dq \rho(q) \left[a\left(\lambda - q, \frac{1}{2}\right) + a\left(\lambda + q, \frac{1}{2}\right) \right], \tag{92}
\end{aligned}$$

$$\begin{aligned}
&2a\left(q, \frac{1}{2}\right) + \frac{1}{G} [a(q, C_a + 1) + a(q, C_b + 1)] = 2\rho(q) + 2\rho^h(q) \\
&\quad + \int d\lambda \sigma(\lambda) \left[a\left(q - \lambda, \frac{1}{2}\right) + a\left(q + \lambda, \frac{1}{2}\right) \right] \\
&\quad + \int d\hat{\lambda} \hat{\sigma}(\hat{\lambda}) \left[a\left(q - \hat{\lambda}, \frac{1}{2}\right) + a\left(q + \hat{\lambda}, \frac{1}{2}\right) \right], \tag{93}
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{G} \left[a\left(\hat{\lambda}, \frac{1}{2}\right) + a\left(\hat{\lambda}, C_a + \frac{1}{2}\right) + a\left(\hat{\lambda}, C_b + \frac{1}{2}\right) - a\left(\hat{\lambda}, C_a - \frac{1}{2}\right) - a\left(\hat{\lambda}, C_b - \frac{1}{2}\right) \right] \\
&\quad + \int dq \rho(q) \left[a\left(\hat{\lambda} - q, \frac{1}{2}\right) + a\left(\hat{\lambda} + q, \frac{1}{2}\right) \right] \\
&= 2\hat{\sigma}(\hat{\lambda}) + 2\hat{\sigma}^h(\hat{\lambda}) + \int d\hat{\lambda}' \hat{\sigma}(\hat{\lambda}') [a(\hat{\lambda} - \hat{\lambda}', 1) + a(\hat{\lambda} + \hat{\lambda}', 1)], \tag{94}
\end{aligned}$$

where $a(\lambda, \eta) \equiv \eta/[\pi(\lambda^2 + \eta^2)]$ with the arbitrary parameter η . The terms with factors $1/G$ in the above three equations describe the finite-size corrections of the system.

B. Properties of Ground State

For the system with N electrons, by using the distributed functions $\sigma(\lambda), \hat{\sigma}(\hat{\lambda})$ and $\rho(q)$, the particle number and magnetization per unit length are given by

$$\begin{aligned}
\frac{N}{G} &= \int dq \rho(q) + 2 \int d\lambda \sigma(\lambda), \\
\frac{S_z}{G} &= \frac{1}{2} \int dq \rho(q) - \int d\lambda \hat{\sigma}(\lambda). \tag{95}
\end{aligned}$$

The energies per unit length have the forms as

$$\frac{E}{G} = -\frac{2N}{G} + 2\pi \int dq \rho(q) a\left(q, \frac{1}{2}\right) + 2\pi \int d\lambda \sigma(\lambda) a(\lambda, 1) \tag{96}$$

for the case of $J = 2$, $V = -1/2$ and

$$\frac{E}{G} = \frac{2N}{G} - 2\pi \int dq \rho(q) a\left(q, \frac{1}{2}\right) - 2\pi \int d\lambda \sigma(\lambda) a(\lambda, 1) \tag{97}$$

for the case of $J = -2$, $V = 1/2$. The relations (92), (93) and (94) become as

$$2a(\lambda, 1) = 2\sigma(\lambda) + 2\sigma^h(\lambda) + \int d\lambda' \sigma(\lambda') [a(\lambda - \lambda', 1) + a(\lambda + \lambda', 1)]$$

$$+ \int dq \rho(q) \left[a \left(\lambda - q, \frac{1}{2} \right) + a \left(\lambda + q, \frac{1}{2} \right) \right], \quad (98)$$

$$\begin{aligned} 2a \left(q, \frac{1}{2} \right) &= 2\rho(q) + 2\rho^h(q) + \int d\lambda \sigma(\lambda) \left[a \left(q - \lambda, \frac{1}{2} \right) + a \left(q + \lambda, \frac{1}{2} \right) \right] \\ &+ \int d\hat{\lambda} \hat{\sigma}(\hat{\lambda}) \left[a \left(q - \hat{\lambda}, \frac{1}{2} \right) + a \left(q + \hat{\lambda}, \frac{1}{2} \right) \right], \end{aligned} \quad (99)$$

$$\begin{aligned} &\int dq \rho(q) \left[a \left(\hat{\lambda} - q, \frac{1}{2} \right) + a \left(\hat{\lambda} + q, \frac{1}{2} \right) \right] \\ &= 2\hat{\sigma}(\hat{\lambda}) + 2\hat{\sigma}^h(\hat{\lambda}) + \int d\hat{\lambda}' \hat{\sigma}(\hat{\lambda}') [a(\hat{\lambda} - \hat{\lambda}', 1) + a(\hat{\lambda} + \hat{\lambda}', 1)], \end{aligned} \quad (100)$$

if we set $G \rightarrow +\infty$. By Fourier transformation of equation (98) we get that

$$\int dq \rho(q) + 2 \int d\lambda \sigma(\lambda) + \int d\lambda \sigma^h(\lambda) = 1,$$

which gives that $N/G = 1 - \int d\lambda \sigma^h(\lambda)$. Owing to $\sigma^h(\lambda) \geq 0$, we have that $N \leq G$, which coincides with the single-occupancy of every site. We assume that there is one particle per lattice site, that is, $N/G = 1$. Then we have $\sigma^h(\lambda) = 0$. Now we consider the case of nonmagnetic $\rho(q) = 0$. The relation (98) turns into

$$a(\lambda, 1) = \sigma(\lambda) + \int d\lambda' \sigma(\lambda') a(\lambda - \lambda', 1). \quad (101)$$

By Fourier transformation of equation (100) we have that $\hat{\sigma}(\hat{\lambda}) = \hat{\sigma}^h(\hat{\lambda}) = 0$, which means that $S_z/G = 0$ and the system is nonmagnetic. From the above relation, we have that

$$\sigma(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega\lambda} \frac{e^{-\frac{|\omega|}{2}}}{2 \cosh \frac{\omega}{2}} d\omega. \quad (102)$$

The interesting thing is that the above expression is exactly same as the integrable narrow-band model with periodic boundary obtained by Schlottmann [22]. In this way, relation (99) reduces to

$$a \left(q, \frac{1}{2} \right) = \rho^h(q) + \int d\lambda \sigma(\lambda) a \left(q - \lambda, \frac{1}{2} \right),$$

and it gives that

$$\rho^h(q) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{-i\omega q}}{2 \cosh \frac{\omega}{2}} d\omega = \begin{cases} \frac{1}{2} \sec h |\pi q|, & \text{for } q \neq 0 \\ \frac{1}{2}, & \text{for } q = 0 \end{cases}. \quad (103)$$

The number M of the down spins is equal to $G/2$. The ground-state energy is $E/G = -2 \ln 2$ for $J = 2$, $V = -1/2$, which has the same value as the one in the periodic boundary condition [22]. It is due to that the impurities located at the both ends cause only the finite-size correction of the ground state energy. For the case of $J = -2$, $V = 1/2$, corresponding to the ferrimagnetic state, we have that

$$\begin{aligned} \rho(q) &= \frac{1}{\pi} \frac{\frac{1}{2}}{q^2 + \frac{1}{4}}, \\ \hat{\sigma}^h(\lambda) &= \frac{1}{\pi} \frac{1}{\lambda^2 + 1}, \quad \hat{\sigma}(\lambda) = 0, \end{aligned} \quad (104)$$

by taking into account of $\sigma(\lambda) = \sigma^h(\lambda) = 0$. Then we have $E/G = 0$.

VI. FINITE-SIZE CORRECTION OF THE GROUND STATE

We assume that the distribution functions $\sigma(\lambda)$, $\rho(q)$ and $\hat{\sigma}(\hat{\lambda})$ are even functions about parameters λ , q and $\hat{\lambda}$, respectively. Then we have the following equations:

$$\begin{aligned} & a(\lambda, 1) + \frac{1}{2G} \left[a\left(\lambda, C_a + \frac{3}{2}\right) + a\left(\lambda, C_a - \frac{1}{2}\right) + a\left(\lambda, C_b + \frac{3}{2}\right) + a\left(\lambda, C_b - \frac{1}{2}\right) \right] \\ &= \frac{1}{2G} a\left(\lambda, \frac{1}{2}\right) + \sigma(\lambda) + \sigma^h(\lambda) + \int d\lambda' \sigma(\lambda') a(\lambda - \lambda', 1) + \int dq \rho(q) a\left(\lambda - q, \frac{1}{2}\right), \end{aligned} \quad (105)$$

$$\begin{aligned} & a\left(q, \frac{1}{2}\right) + \frac{1}{2G} [a(q, C_a + 1) + a(q, C_b + 1)] \\ &= \rho(q) + \rho^h(q) + \int d\lambda \sigma(\lambda) a\left(q - \lambda, \frac{1}{2}\right) + \int d\hat{\lambda} \hat{\sigma}(\hat{\lambda}) a\left(q - \hat{\lambda}, \frac{1}{2}\right), \end{aligned} \quad (106)$$

$$\begin{aligned} & \frac{1}{2G} \left[a\left(\hat{\lambda}, \frac{1}{2}\right) + a\left(\hat{\lambda}, C_a + \frac{1}{2}\right) + a\left(\hat{\lambda}, C_b + \frac{1}{2}\right) - a\left(\hat{\lambda}, C_a - \frac{1}{2}\right) - a\left(\hat{\lambda}, C_b - \frac{1}{2}\right) \right] \\ &+ \int dq \rho(q) a\left(\hat{\lambda} - q, \frac{1}{2}\right) \\ &= \hat{\sigma}(\hat{\lambda}) + \hat{\sigma}^h(\hat{\lambda}) + \int d\hat{\lambda}' \hat{\sigma}(\hat{\lambda}') a(\hat{\lambda} - \hat{\lambda}', 1), \end{aligned} \quad (107)$$

from equations (92), (93) and (94), where $a(\lambda, \eta) \equiv \eta/[\pi(\lambda^2 + \eta^2)]$ with the arbitrary real parameter η . The terms with factors $1/G$ in the above three equations describe the finite-size corrections of the system. The energies of the system can be described by

$$\frac{E}{G} = \mp \frac{2N}{G} \pm 2\pi \left[\int dq \rho(q) a\left(q, \frac{1}{2}\right) + \int d\lambda \sigma(\lambda) a(\lambda, 1) \right] \quad (108)$$

for $J = \pm 2$, $V = \mp 1/2$, respectively. Setting

$$S_\eta \equiv \text{sign}(\eta) = \begin{cases} 1, & \eta > 0 \\ -1, & \eta < 0 \\ 0, & \eta = 0 \end{cases}, \quad (109)$$

we have that

$$\tilde{a}(\omega, \eta) = S_\eta \exp(-|\omega\eta|). \quad (110)$$

By Fourier transformation of equation (105), we have

$$\tilde{\sigma}^h(0) = \frac{1}{2G} \left[S_{C_a + \frac{3}{2}} + S_{C_a - \frac{1}{2}} + S_{C_b + \frac{3}{2}} + S_{C_b - \frac{1}{2}} - 1 \right]$$

for $N/G = 1$. By letting

$$b(\lambda) = a\left(\lambda, C_a + \frac{3}{2}\right) + a\left(\lambda, C_a - \frac{1}{2}\right) + a\left(\lambda, C_b + \frac{3}{2}\right) + a\left(\lambda, C_b - \frac{1}{2}\right) - a\left(\lambda, \frac{1}{2}\right), \quad (111)$$

we have that

$$\int dq \rho(q) a\left(q, \frac{1}{2}\right) + \int d\lambda \sigma(\lambda) a(\lambda, 1) = a(0, 1) + \frac{b(0)}{2G} - \sigma(0) - \sigma^h(0).$$

We set $\sigma^h(\lambda) \equiv 0$. The Kondo coupling constants C_a and C_b should be in the ranges (i) $C_a > 1/2$, $C_b = -3/2$; (ii) $C_a = 1/2$, $1/2 > C_b > -3/2$; (iii) $1/2 > C_a > -3/2$, $C_b = 1/2$; (iv) $C_a = -3/2$, $C_b > 1/2$. For the case of $J = 2$, $V = -1/2$, the ground state energy can be written down as the form

$$\frac{E}{G} = \frac{\pi}{G} b(0) - 2\pi\sigma(0), \quad (112)$$

and $\sigma(0)$ should take its largest value. Then we set $\rho(q) = 0$ and obtain that

$$2\pi\sigma(0) = 2\ln 2 + \frac{1}{2G} \int_{-\infty}^{+\infty} \frac{\tilde{b}(\omega)}{1 + \exp(-|\omega|)} d\omega \quad (113)$$

where

$$\begin{aligned} \tilde{b}(\omega) = & S_{C_a + \frac{3}{2}} \exp\left[-\left|\omega\left(C_a + \frac{3}{2}\right)\right|\right] + S_{C_a - \frac{1}{2}} \exp\left[-\left|\omega\left(C_a - \frac{1}{2}\right)\right|\right] \\ & + S_{C_b + \frac{3}{2}} \exp\left[-\left|\omega\left(C_b + \frac{3}{2}\right)\right|\right] + S_{C_b - \frac{1}{2}} \exp\left[-\left|\omega\left(C_b - \frac{1}{2}\right)\right|\right] - \exp\left(-\left|\frac{\omega}{2}\right|\right). \end{aligned}$$

Therefore, the finite-size correction of the ground-state energy due to impurities is

$$E' = \frac{8(2C_a^2 + 3C_a + 2)}{(2C_a + 3)(4C_a^2 - 1)} - \frac{3}{2} - \ln 2 + \frac{\pi}{2} - 2\beta\left(\frac{2C_a - 1}{2}\right) \quad (114)$$

when $C_a > 1/2$ and $C_b = -3/2$, where β is defined by $\beta(x) = \frac{1}{2} [\psi(\frac{x+1}{2}) - \psi(\frac{x}{2})]$ and $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$. By taking account of $C_a > 1/2$, then $J_a < 0$, we have $\hat{\sigma}(\hat{\lambda}) = 0$ for the ground state. From relations (105), (106) and (107), we get that

$$\hat{\sigma}^h(\hat{\lambda}) = \frac{1}{2G} \left[a\left(\hat{\lambda}, \frac{1}{2}\right) + a\left(\hat{\lambda}, C_a + \frac{1}{2}\right) + a\left(\hat{\lambda}, -1\right) - a\left(\hat{\lambda}, C_a - \frac{1}{2}\right) - a\left(\hat{\lambda}, -2\right) \right], \quad (115)$$

$$\rho^h(0) = \frac{1}{2} + \frac{1}{2\pi G} \left[\frac{1}{C_a} - \frac{10}{3} + \ln 2 + \frac{\pi}{2} - 2\beta(C_a) \right], \quad (116)$$

$$\rho^h(q) = \frac{1}{2} \sec h |\pi q| + \frac{1}{2G} \left[a(q, C_a + 1) - a\left(q, \frac{1}{2}\right) \right] - \frac{1}{4\pi G} \int_0^{+\infty} \frac{\tilde{b}(\omega) \cos(\omega q)}{\cosh \frac{\omega}{2}} d\omega \quad (117)$$

for $q \neq 0$. When $C_a = 1/2$, $-3/2 < C_b < 1/2$, from relation (113), we have that

$$2\pi\sigma(0) = 2\ln 2 + \frac{1}{2G} \left\{ 2(1 - \ln 2) - \pi + 2 \left[\beta\left(\frac{2C_b + 3}{2}\right) - \beta\left(\frac{1 - 2C_b}{2}\right) \right] \right\}. \quad (118)$$

Then, the finite-size correction of the ground state has the form

$$\begin{aligned} E' = & \frac{\pi}{2} \left[\cot\left(\frac{1}{4}\pi + \frac{1}{2}\pi C_b\right) + \tan\left(\frac{1}{4}\pi + \frac{1}{2}\pi C_b\right) + 1 \right] \\ & + \ln 2 - \frac{1}{2} \frac{44C_b^2 - 26C_b - 35 + 40C_b^3}{(2C_b + 3)(2C_b - 1)(2C_b + 1)}. \end{aligned} \quad (119)$$

By taking account of $J_b > 0$, we have $\hat{\sigma}^h(\hat{\lambda}) = 0$. From relations (105), (106) and (107), we obtain that

$$\begin{aligned} \hat{\sigma}(\hat{\lambda}) = & \frac{1}{4\pi G} \int_0^{+\infty} d\omega \frac{\cos(\omega\lambda)}{\cosh \frac{\omega}{2}} \left\{ 1 + \exp\left(-\left|\frac{\omega}{2}\right|\right) + \right. \\ & \left. + S_{C_b+\frac{1}{2}} \exp\left[\left|\frac{\omega}{2}\right| - \left|\omega\left(C_b + \frac{1}{2}\right)\right|\right] \right\}, \end{aligned} \quad (120)$$

$$\begin{aligned} \rho^h(q) = & \frac{1}{2} \sec h |\pi q| + \frac{1}{2G} \left[a\left(q, \frac{3}{2}\right) + a(q, C_b + 1) \right] \\ & - \frac{1}{4\pi G} \int_0^{+\infty} d\omega \frac{\cos(\omega q)}{\cosh \frac{\omega}{2}} \left\{ \exp(-|2\omega|) + \exp(-|\omega|) + \exp\left[-\left|\omega\left(C_b + \frac{3}{2}\right)\right|\right] \right. \\ & \left. + S_{C_b+\frac{1}{2}} \exp\left[-\left|\omega\left(C_b + \frac{1}{2}\right)\right|\right] \right\} \end{aligned} \quad (121)$$

for $q \neq 0$ and

$$\rho^h(0) = \begin{cases} \frac{1}{2} & \text{for } C_b = -1 \\ \frac{1}{2} + \frac{1}{2\pi G} \left\{ \frac{1}{C_b+1} + \beta(C_b+2) + S_{C_b-\frac{1}{2}} \beta\left(\frac{|2C_b+1|+1}{2}\right) \right\} & \text{for } C_b \neq -1 \end{cases} \quad (122)$$

The cases of $1/2 > C_a > -3/2$, $C_b = 1/2$; $C_a = -3/2$, $C_b > 1/2$ have the similar expressions. When $J = -2$, $V = 1/2$, by similar discussions, the finite-size correction of the ground-state energy can be written down as

$$E' = \frac{5}{2} - \frac{4(2C_a+1)}{(2C_a-1)(2C_a+3)} \quad (123)$$

for $C_a > 1/2$, $C_b = -3/2$ and

$$E' = \frac{3}{2} - \frac{4(2C_b+1)}{(2C_b-1)(2C_b+3)} \quad (124)$$

for $C_a = 1/2$, $1/2 > C_b > -3/2$.

As the conclusions, an integrable model in one dimension is constructed from $t - J$ model where two magnetic impurities are coupled to the system. It describe the behavior of the strong correlation electrons with Kondo problem. The spectrums of the system is not linear. The boundary R matrix depends on the spin and rapidity of the particle and satisfies the reflecting factorizable condition. The Hamiltonian of the model is diagonalized exactly by the Bethe-Ansatz method. The integral equations are derived with the complex ‘‘rapidities’’ q which describe the bound states in the system. The properties of the ground state are discussed and the finite-size corrections of the ground-state energies are obtained due to the couplings of the magnetic impurities.

-
- [1] J. Kondo Prog. Theor. Phys. 32 (1964) 37; Solid State Phys. 23 (1969) 183.
 - [2] J.M. Luttinger, J. Math. Phys. 4 (1963) 1154; D. C. Mattis and E.H. Lieb, J. Math. Phys. 6 (1965) 304.
 - [3] P. Fröjdh and H. Johannesson, Phys. Rev. Lett. 75 (1995) 300; F. D. Haldane, Ibid., 45 (1980) 1358.
 - [4] X. G. Wen, Phys. Rev. B44 (1991) 5708.
 - [5] C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. 68 (1992) 1220; Phys. Rev. B 46 (1992) 15233.
 - [6] D.-H. Lee and J. Toner, Phys. Rev. Lett. 69 (1992) 3378.
 - [7] A. Furusaki and N. Nagaosa, Phys. Rev. Lett. 72 (1994) 892.
 - [8] P. Fröjdh and H. Johannesson, Phys. Rev. Lett. 75 (1995) 300.
 - [9] Y. Wang, J. Voigt and F. -C. Pu, Phys. Rev. B 54 (1996) 8491.
 - [10] X. Wang, cond-mat/9705302.
 - [11] Y. Oreg and M. Finkel'stein, cond-mat/9707162.
 - [12] G. Bedürftig, F.H.L. Eßler, H. Frahm, Phys. Rev. Lett., 69 (1996) 5098; Nucl. Phys. B 489 (1997) 697.
 - [13] N. Andrei and H. Johannesson, Phys. Lett. A 100 (1984) 108.
 - [14] Y.P. Wang and J. Voigt, Phys. Rev. Lett. 77 (1996) 4934.
 - [15] Y.P. Wang, J.H. Dai, Z.N. Hu and F.C. Pu, Phys. Rev. Lett. 79 (1997) 1901.
 - [16] Y.-P. Wang and P.-C. Pu, Preprint.
 - [17] G. Jüttner, A. Klümper, J. Suzuki, Nucl. Phys. B 487 (1997) 650.
 - [18] Y.-K. Zhou and M.T. Batchelor, Nucl. Phys. B 490 (1997) 576.
 - [19] A.J. Bracken, M.D. Gould, J.R. Links and Y.-Z. Zhang, Phys. Rev. Lett. 74 (1995) 2768.
 - [20] F.H.L. Eßler, V.E. Korepin and K. Schoutens, Phys. Rev. Lett. 68 (1992) 2960.
 - [21] E. K. Sklyanin, J. Phys. A 21 (1988) 2375; C. Destri and H. J. de Vega, Nucl. Phys. B 385 (1992) 361; B 374 (1992) 692.
 - [22] P. Schlottmann, Phys. Rev. B 36 (1987) 5177.
 - [23] P. W. Anderson, Science 235 (1987) 1196.
 - [24] P.-A. Bares, G. Blatter, and M. Ogata, Phys. Rev. B 44 (1991) 130.
 - [25] A. Foerster and M. Karowski, Nucl. Phys. B 386 (1993) 611; 408 (1993) 512.
 - [26] F.H. L. Essler and V.E. Korepin, Phys. Rev. B 46 (1992) 9147.
 - [27] H.J. de Vega and A. González-Ruiz, Nucl. Phys. B 417 (1994) 553.
 - [28] H. Schultz, J. Phys. C 18 (1985) 581.
 - [29] M. Takahashi, Prog. Theor. Phys. 46 (1971) 401.